Applications of Covariance Structure Modeling in Psychology: Cause for Concern?

Steven J. Breckler
Johns Hopkins University

Methods of covariance structure modeling are frequently applied in psychological research. These methods merge the logic of confirmatory factor analysis, multiple regression, and path analysis within a single data analytic framework. Among the many applications are estimation of disattenuated correlation and regression coefficients, evaluation of multitrait-multimethod matrices, and assessment of hypothesized causal structures. Shortcomings of these methods are commonly acknowledged in the mathematical literature and in textbooks. Nevertheless, serious flaws remain in many published applications. For example, it is rarely noted that the fit of a favored model is identical for a potentially large number of equivalent models. A review of the personality and social psychology literature illustrates the nature of this and other problems in reported applications of covariance structure models.

A principal goal of experimentation in psychology is to provide a basis for inferring causation. Among the tools used to achieve this goal are the active manipulation and control of independent variables, random assignment to experimental treatments, and appropriate methods of data analysis. Causal inferences are difficult to support without true experimentation. Nevertheless, social and behavioral scientists often make such inferences in the context of nonexperimental and quasi-experimental research (cf. Blalock, 1961; Cook & Campbell, 1979). A variety of sophisticated methods for multivariate data analysis have been developed and used in these situations (e.g., Kenny, 1979), many of which fall into the general category of covariance structure modeling.

Covariance structure modeling can be used to test whether a hypothesized causal structure is consistent or inconsistent with the data. Applications of covariance structure modeling are still rare, but several criteria suggest rapidly increasing use (cf. Bentler, 1986). Published accounts have increased markedly since 1979. Recent textbooks on multivariate analysis have included chapters on covariance structure modeling (e.g., Bernstein, 1988; Pedhazur, 1982), and several textbooks have been devoted entirely to the topic (e.g., Hayduk, 1987; Loehlin, 1987; Long, 1983).

The purposes of this article are to show how covariance structure modeling is currently being applied in two related areas of research (personality and social psychology), and to document potential flaws in many published applications. The personality and social psychology literature is used primarily for illustrative purposes. The major points of this article apply with equal force to other areas of research in which covariance structure modeling has been used, including developmental and educational psychology, sociology, political science, economics, marketing, and consumer behavior.

The first section of this article provides a brief overview of one popular representation of a covariance structure model: the LISREL model (Jöreskog & Sörbom, 1984). This section defines the basic elements of the LISREL model and identifies common problems associated with this and other models for the analysis of covariance structures. The second section briefly summarizes the nature of published applications in several representative personality and social psychology journals and illustrates the method with an example from the social psychological literature. The third section examines how problems associated with covariance structure modeling have been addressed in published accounts and how failures to recognize such problems often lead to misleading conclusions. The final section offers several recommendations for reporting the results of covariance structure modeling.

The Covariance Structure Model

Covariance structure modeling merges the logic of confirmatory factor analysis, multiple regression, and path analysis within a single data analytic framework (cf. Bentler, 1980). Several models for the analysis of covariance structures have been developed (e.g., Bentler, 1985; Bentler & Weeks, 1980; Browne, 1984; Jöreskog, 1978; McArdle & McDonald, 1984; Lee & Jennrich, 1984), but the majority of applications have used the LISREL\(^1\) (Jöreskog & Sörbom, 1984) representation. Several textbooks provide a detailed description of the LISREL model.

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\(^{1}\) LISREL (Linear Structural Relations) is the name of a computer program used for covariance structure modeling (Jöreskog & Sörbom, 1984).

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Correspondence concerning this article should be addressed to Steven J. Breckler, Department of Psychology, Johns Hopkins University, Baltimore, Maryland 21218.
Covariance structure modeling rests on the logic of covariance structure modeling, which defines hypothetical latent variables (LVs) in terms of observed measured variables (MVs), and the structural model, which defines relations among the LVs. A distinction is also made between exogenous (independent) variables and endogenous (dependent) variables. All variables therefore fall into one of four sets: \(q\) exogenous MVs \((x_1, x_2, \ldots, x_q)\), \(p\) endogenous MVs \((y_1, y_2, \ldots, y_p)\), \(n\) exogenous LVs \((\xi_1, \xi_2, \ldots, \xi_n)\), and \(m\) endogenous LVs \((\eta_1, \eta_2, \ldots, \eta_m)\).

Two matrix equations are used to represent the measurement portion of the LISREL model:

\[
X = \Lambda_x \xi + \delta, \quad (1)
\]

and

\[
Y = \Lambda_y \eta + \epsilon. \quad (2)
\]

In Equation 1, \(X\) is a \(q \times 1\) vector of exogenous MVs; \(\Lambda_x\) is a \(q \times n\) matrix of coefficients (factor loadings) that indicate the influence of the exogenous LVs on the exogenous MVs; \(\delta\) is a \(q \times 1\) vector of errors in measurement for the exogenous MVs. In Equation 2, \(Y\) is a \(p \times 1\) vector of endogenous MVs; \(\Lambda_y\) is a \(p \times m\) matrix of coefficients (factor loadings) that indicate the influence of the endogenous LVs on the endogenous MVs; \(\eta\) is a \(m \times 1\) vector of endogenous LVs; and \(\epsilon\) is a \(p \times 1\) vector of errors in measurement for the endogenous MVs.

The matrix equation for the structural portion of the model is

\[
\eta = B \eta + \Gamma \xi + \xi. \quad (3)
\]

In Equation 3, \(B\) is a \(m \times m\) matrix of coefficients that indicate the influence of endogenous LVs on other endogenous LVs; \(\Gamma\) is a \(m \times n\) matrix of coefficients that indicate the influence of exogenous LVs on endogenous LVs; \(\xi\) is a \(m \times 1\) vector of errors in prediction for the \(m\) endogenous LV equations. LISREL defines four additional matrices: \(\Theta\) is a \(q \times q\) matrix of covariances among the errors in measurement for Equation 1; \(\Theta_0\) is a \(p \times p\) matrix of covariances among the errors in measurement for Equation 2; \(\Phi\) is a \(n \times n\) matrix of covariances among the exogenous LVs \((\xi_1, \xi_2, \ldots, \xi_n)\); and \(\Psi\) is a \(m \times m\) matrix of covariances among the errors in prediction \((\xi_1, \xi_2, \ldots, \xi_m)\).

The population covariance matrix \(\Sigma\) is defined as a function of eight parameter matrices \((\Lambda_x, \Lambda_y, \Theta_0, \Theta, \Phi, B, \Gamma, \Psi)\). A given theoretical model is represented by specifying a pattern of fixed and free (estimated) elements in each of the eight parameter matrices. The matrix of observed covariances \((S)\) is then used to estimate values for the free parameters that best reproduce the data. Although maximum likelihood estimates are most frequently used, other methods for parameter estimation are available (e.g., least squares).

Covariance structure modeling rests on the logic of confirmatory analysis. A theoretical model must be specified on a priori grounds, such that it contains fewer free parameters than observed data points (the number of variances and covariances). A model gains support when the observed data can be closely reproduced by a set of estimated parameters, subject to the constraints imposed on the parameter matrices. Thus, covariance structure modeling is used to test whether a given theoretical model is consistent or inconsistent with the data.

### Identification

Identification refers to whether the individual parameters in a model and the model as a whole have unique solutions. If a parameter is not identified, its estimates will be arbitrary and uninterpretable. For example, in the equation \(x + 3 = 5\), the unknown quantity \(x\) is identified because a unique solution for \(x\) exists. However, in the equation \(a + b = 5\), two unknown quantities are involved; without further information (e.g., other equations containing \(a\) and \(b\)), a unique solution for the values of \(a\) and \(b\) does not exist (for more, see Duncan, 1975, or Hayduk, 1987).

Unfortunately, computer programs often compute plausible parameter estimates even when some of the parameters are not fully identified. It can be difficult to detect such problems, especially in models involving large numbers of parameters. Identification can always be determined by solving equations that express each parameter in terms of known quantities, but this process can be tedious and difficult. Other criteria can sometimes be used, but they apply only to particular forms of models (see Long, 1983). An empirical check on identification can also be performed by using the LISREL computer program (see Jöreskog & Sörbom, 1984, p. 1.24).

### Assessment of Fit

Perhaps the greatest practical concern is determining how well a model reproduces or fits the data. Goodness of fit can be assessed in many ways. Practitioners are generally advised to examine multiple fit criteria rather than to rely on a single statistic. Proper assessment of a model's fit involves evaluation of the entire model, each equation within the model, and the individual parameter estimates.

Several indices are available for determining the global fit of a model. The most commonly reported fit index is the chi-square statistic, which is available when maximum likelihood estimation is done. However, several difficulties are associated with using the chi-square value as a test statistic (cf. Satorra & Saris, 1985). For example, well-fitting models are ones that produce a small chi-square, or a failure to reject the null hypothesis. The chi-square test can also be sensitive to small differences between observed and estimated data, especially when the sample size is large. Several supplemental fit indices have been proposed (see Hoelter, 1983). For example, Bentler and Bonett (1980) have developed normed \((\Delta)\) and non-normed \((\rho)\) indices that indicate the fit of a theoretical model relative to a logical worst case (null) model.

Another global fit index is the root mean square residual (RMR), which reflects the average deviation between observed covariances \((S)\) and their estimates. The scale of an RMR depends on the data being analyzed. When correlations are used, the RMR will be in correlation units and will be readily inter-
interpretable. However, when covariances are used, the RMR will be expressed in covariance units that may be difficult to interpret.

The global fit of a given model can also be evaluated by comparing it with a theoretically competing model. This method is most useful when one model is hierarchically nested within the other (i.e., when one contains the same set of parameters as the other, plus some additional parameters). In these cases, a difference in chi-squares for the two models is itself a chi-square statistic that indicates whether the model with more free parameters is better able to reproduce the data.

In addition to testing the global fit of an entire model, it is important to evaluate the fit of individual equations within a model. For this purpose, a squared multiple correlation can be computed for each structural equation in the model. These values indicate the proportion of variance in each latent endogenous variable accounted for by the equation.

The reliability of individual parameter estimates is also of critical importance. Approximate $t$ values can be computed for each parameter to test the null hypothesis that its value is zero. The global fit of a model may be very good even when one or more individual parameters is not reliably different than zero.

Other Issues and Problems

The problems of identification and assessment of fit are given considerable attention in textbook treatments. However, logical problems of the sort discussed by Cliff (1983) are not commonly addressed. One problem is that "acceptable" models are those that the data fail to disconfirm; when a model is not disconfirmed, many other equally fitting models are also not disconfirmed. Another problem occurs when the same set of data is used to both modify a model and evaluate its fit, a practice that undermines the entire logic of confirmatory analysis. Third, although correlational data may be consistent with hypothesized causal relations, they do not provide a sufficient basis for establishing causation (cf. Baumrind, 1983; Cook & Campbell, 1979). Finally, the invention of LVs to explain an observed pattern of correlations does not guarantee that the invented LVs truly exist or that they are understood (the nominalistic fallacy).

Equivalent models. A well-fitting model is one that closely reproduces the observed data. However, even when a model is found to be consistent with the data, it is almost always the case that alternative models will be equally consistent. Although this situation is sometimes acknowledged in textbooks (e.g., Loehlin, 1987; Pedhazur, 1982), it is not always so (e.g., Hayduk, 1987; Long, 1983). Stelzl (1986) has developed several rules that help to generate a family of equivalent models. Two models are defined as equivalent when they are associated with identical fit functions (e.g., maximum likelihood loss functions). Two equivalent models will account for the same observed covariances equally well, every global goodness of fit statistic will be identical for the two models ($\chi^2$, $p$, $\Delta$, RMR, etc.). The results from covariance structure modeling simply cannot be used to distinguish the global fit of two equivalent models. However, two equivalent models can differ in the values estimated for individual parameters, thereby providing some basis for distinguishing between them.

Other well-fitting models. If a model is found to provide a good global fit, it is very likely that many other models will also provide a good (if not better) fit to the data. It can be difficult to construct even a small number of such alternative models, especially when many of them are not easily derived from substantive theory. The TETRAD computer program (Glymour, Scheines, Spirtes, & Kelly, 1987) was developed to help in the search for alternative well-fitting models. Beginning with the data, a starting model, and user-specified constraints on the search process, the TETRAD algorithm can locate alternative models that do a relatively good job in reproducing the data. The majority of models produced by the TETRAD program are not likely to be readily interpretable. Nevertheless, the program may produce theoretically viable alternative models that might otherwise have been overlooked.

Model modification. A priori theoretical models frequently do not provide an adequate fit to the data. When this happens, the original model may be modified to improve its fit. Modifications to the measurement portion of a model can be made by (a) changing factor loadings ($\lambda$s) from fixed to free or from free to fixed, (b) allowing or constraining correlations among measurement errors ($\delta$s, $\epsilon$s), and (c) allowing or constraining correlations among the exogenous LVs ($\gamma$s). The structural model can be modified by (a) changing path coefficients ($\gamma$s, $\delta$s) from fixed to free or from free to fixed, and (b) allowing or constraining correlations among the errors in equations ($\delta$s).

Modifications can often be derived on theoretical grounds, but the more common practice is to modify models on the basis of the data. For example, the LISKEL computer program (Joreskog & Sorbom, 1984) calculates a "modification index" for every fixed parameter in a model. The modification index reflects the minimum reduction in the chi-square statistic if the parameter is changed from fixed to free.

The goal of model modification is to develop a model that is consistent with the data. However, there is no guarantee that the process of model modification (or specification searching) will lead to the population model, and in some cases it is unlikely to do so (MacCallum, 1986). Investigators are urged to interpret the results of a specification search with caution and to cross-validate the model whenever possible (Cudeck & Browne, 1983).

Causal inferences. Covariance structure modeling goes by many names, one of which is causal modeling (e.g., Bentler, 1980). Indeed, covariance structure modeling can be used to test whether a hypothesized causal structure is consistent or inconsistent with the data. However, covariance structure modeling does not provide a sufficient basis for drawing causal inferences (cf. Baumrind, 1983). If one model is supported, so too are all of its equivalent models. Causal inferences ultimately depend on criteria that lie outside the data analytic system, such as manipulation and control of independent variables (cf. Cliff, 1983). The term causal modeling is a misnomer and should not be so casually applied.

Computer Programs

Several computer programs are available to estimate covariance structure models. The LISREL computer program (Joreskog & Sorbom, 1984) was the first widely used program. A more recent computer program, EQS (Bentler, 1985), repre-
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sents the covariance structure model in a different way and can be used to estimate a broader class of models. Programs originally developed for other purposes can also be used to estimate structural equation models (e.g., Lee & Jennrich, 1984). Such programs are available for microcomputers and in commercial statistical packages. Thus, software for covariance structure modeling is readily accessible.

Assumptions

Maximum likelihood estimation of the parameters in a covariance structure model assumes that the variables have a multivariate normal distribution in the population. Distribution-free methods for parameter estimation are also available, but the maximum likelihood method is most commonly used. Several of the textbooks provide a good discussion of the multivariate normal assumption (e.g., Hayduk, 1987). When the raw data are available, the EQS (Bentler, 1985) computer program will compute several statistics developed by Mardia (1974) for testing this assumption. Jöreskog and Sörbom (1984) have warned that standard errors must be interpreted with caution when the normality assumption has been violated.

The development of goodness of fit statistics and standard errors is based on asymptotic normal theory. This means that large samples are typically required for proper interpretation of the chi-square global fit statistic and to justify the use of standard errors in testing hypotheses about individual parameter estimates. Research is only beginning to examine the consequences of small sample size on estimating a covariance structure model (e.g., Anderson & Gerbing, 1984; Gerbing & Anderson, 1983; Geweke & Singleton, 1980). Tanaka (1987) provided a good overview of the sample size issue and also suggested the application of alternative fit indices when sample size is small.

Applications in Personality and Social Psychology

Nature of Applications

Covariance structure modeling is being used with increasing frequency in personality and social psychological research. During the period 1977–1987, 72 applications of covariance structure modeling were reported in four representative journals (Journal of Personality and Social Psychology [63 articles], Journal of Experimental Social Psychology [5 articles], Personality and Social Psychology Bulletin [2 articles], and Psychological Review [2 articles]). Figure 1 shows the number of articles in which covariance structure models were used during each of the 11 years covered by this review. A list of the articles appears in the appendix.

Examination of Figure 1 shows that the Journal of Personality and Social Psychology accounts for the majority of published applications within this set of journals. Published accounts of covariance structure analysis were rare before 1979, presumably because computer programs were not widely available. One indication that covariance structure modeling was just being introduced in 1979 was the publication of the Bentler and Speckart (1979) article in Psychological Review. Although that article contributed to theoretical conceptions of the attitude–behavior relation, its primary impact was the introduction of covariance structure modeling to the social psychology literature. The number of published applications appears to be growing over the period 1979–1987, but the increasing trend is not yet a reliable one.

Covariance structure modeling has been used in personality and social psychology to estimate disattenuated correlation and regression coefficients (e.g., Breckler, 1984; Hoelter, 1985), to evaluate multitrait-multimethod matrices (e.g., Howard, 1987; Russell, McAuley, & Tarico, 1987), to test second-order factor models (e.g., Judd & Krosnick, 1982; Newcomb & Bentler, 1986), and to assess hypothesized causal structures (e.g., Bentler & Speckart, 1981; Bohrnstedt & Felson, 1983).

Of the 72 articles included in this review, 29 focused solely on measurement models (i.e., confirmatory factor analysis). Such applications are often used to estimate correlations among theoretical constructs (the latent variables). Because LVs are conceived as error free, correlations among them (contained in the Φ matrix) can be interpreted as having been corrected for attenuation due to unreliability in their measured variables (cf. Allen & Yen, 1979). An example is Breckler's (1984) evaluation of the tripartite (affect–behavior–cognition) model of attitude structure.

The majority of analyses (44) used cross-sectional data; a smaller number (21) used longitudinal data, and 7 used data collected in the context of an experiment. Among the analyses based on cross-sectional data, most focused on measurement models (26). However, a substantial number of studies (18) used cross-sectional data to estimate structural models, even though cross-sectional data collected in a nonexperimental context do not provide a strong basis for developing prediction equations or for inferring causation. Of the 21 articles using longitudinal data, 20 analyzed complete structural models and 1 focused solely on measurement models (Newcomb & Bentler, 1986). Although causal inferences may sometimes be justified in longitudinal contexts, such designs do not always provide a sufficient basis for drawing causal inferences. Nevertheless, longitudinal designs are better suited for developing prediction equations than are cross-sectional designs.

An Illustration

A study reported by Fredricks and Dossett (1983) illustrates the basic application of covariance structure modeling. This analysis was selected because it provides a typical example of a complete covariance structure model. It also suffers from many of the problems described in the previous section.

Fredricks and Dossett (1983) used covariance structure modeling to evaluate Fishbein and Ajzen's (1975) model of attitude–behavior relations. An important component of this analysis was a comparison between the original model and several modifications of it reported by Bentler and Speckart (1979). These models appear to be good candidates for covariance structure modeling because they are very explicit in both the measurement and the structural portions of the model.

According to the model of reasoned action (Fishbein, 1967; Fishbein & Ajzen, 1975), target behaviors (TB) are directly determined by behavioral intentions (BI), which in turn are directly determined by attitudes toward the behaviors (AACT) and by subjective norms (SN). Bentler and Speckart (1979)
modified this model by adding prior behaviors (PB) as a direct determinant of behavioral intentions and of target behaviors, and by adding a direct link between attitudes toward the behaviors and target behaviors.

A path diagram representing the final model favored by Fredricks and Dossett (1983) is shown in Figure 2. Only the latent variables are shown in the diagram. In addition to the directed paths, the three exogenous LVs (AACT, SN, and PB) are intercorrelated. The behavioral domain investigated by Fredricks and Dossett was class attendance. Two measures of AACT ($X_1$ and $X_2$), two measures of SN ($X_3$, $X_4$), three measures of PB ($X_5$, $X_6$, $X_7$), two measures of BI ($Y_1$, $Y_2$), and two measures of TB ($Y_3$, $Y_4$) were collected in a longitudinal context.

The measurement model matrix equation for the exogenous MVs is

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ \lambda_2 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & \lambda_4 & 0 \\ 0 & 0 & \lambda_5 \\ 0 & 0 & \lambda_6 \\ 0 & 0 & \lambda_7 \\ \Lambda_x \end{bmatrix} \begin{bmatrix} AACT \\ SN \\ PB \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \end{bmatrix},$$

(4)

The $\Lambda_x$ matrix contains seven parameters to be estimated from the data; the remaining entries are fixed to zero. The measurement errors ($\delta$s) are not directly estimated; instead their variances (the diagonal entries of $\delta$s) are estimated from the data. Intercorrelations among the three exogenous LVs are represented in the $\Phi$ matrix:

$$\Phi = \begin{bmatrix} 1 & \phi_1 & \phi_2 \\ \phi_1 & 1 & . \\ \phi_2 & . & 1 \end{bmatrix}. \quad (5)$$

The diagonal entries of $\Phi$ are fixed to unity. This is done for three reasons. First, the off-diagonal elements of $\Phi$ can be directly interpreted as correlations. Second, a metric for the three exogenous LVs is established by setting their variances equal to one. Third, the constraints help to make the model identified.

The measurement model matrix equation for the endogenous MVs is

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \lambda_8 \\ \lambda_8 & 0 & 1 & . \\ 0 & \lambda_9 & 0 & \lambda_3 \\ 0 & 0 & \lambda_3 & 0 \end{bmatrix} \begin{bmatrix} BI \\ TB \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \end{bmatrix}. \quad (6)$$

In Equation 6, the $\Lambda_y$ matrix has two free parameters, two entries fixed to one, and the remaining entries fixed to zero. The entries fixed to one are required to establish a metric for the endogenous LVs. The errors in measurement ($\epsilon$s) are not di-

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2 A metric was established for the exogenous LVs by fixing diagonals of $\Phi$ to 1, rather than fixing entries in $\Lambda_x$ to 1. Either method is acceptable for establishing a metric for exogenous LVs. However, variances of the endogenous LVs are not directly estimated in the LISREL model, so the method of fixing entries in $\Lambda_x$ was used. Other methods can be used (e.g., by fixing some entries in the diagonal of $\delta$), but the ones used here are the most common.
Figure 2. A covariance structure model of attitude–behavior relations.

rectly estimated; instead, their variances (the diagonal entries of $\Theta$) are estimated from the data.

Finally, the structural model matrix equation is

$$
\begin{bmatrix}
BI \\
TB
\end{bmatrix}
= \begin{bmatrix}
0 & 0 \\
\beta & 0
\end{bmatrix}
\begin{bmatrix}
BI \\
TB
\end{bmatrix}

\eta
= B \eta
+ \begin{bmatrix}
\gamma_1 & \gamma_2 & \gamma_3 \\
0 & 0 & \gamma_4
\end{bmatrix}
\begin{bmatrix}
AACT \\
SN \\
PB
\end{bmatrix}
+ \begin{bmatrix}
\xi_1 \\
\xi
\end{bmatrix}
(7)

Equation 7 contains five path coefficients to be estimated from the data ($\beta, \gamma_1, \gamma_2, \gamma_3,$ and $\gamma_4$). The errors in prediction ($\psi$) are not directly estimated; instead, their variances (the diagonal entries of $\psi$) are estimated from the data.

This model provided a relatively good overall fit for the data collected by Fredricks and Dossett (1983), $\chi^2(36, N = 236) = 55.6, p > .01$. The supplemental goodness of fit indices $\rho$ and $\Delta$ were, respectively, .96 and .94. Several theoretically competing models were also tested; the model shown in Figure 2 was preferred on the basis of its fit and parsimony. However, inspection of the individual parameter estimates revealed that the factor correlation between SN and PB ($\phi_3$) and two of the path coefficients ($\gamma_2$ and $\beta$) were not reliably different than zero. These results suggest that the model (as shown in Figure 2) may not provide the most parsimonious fit to the data.

Cause for Concern?

Problems associated with covariance structure modeling receive considerable attention in the technical literature. In this section, the personality and social psychology literature is reviewed to learn whether common problems have been acknowledged or recognized in published applications.

Violation of Assumptions

The assumption of multivariate normality was acknowledged in 14 of the 72 articles, and only 7 articles considered whether the assumption had actually been violated. Six of the latter articles used distribution-free methods of parameter estimation to compensate for probable violations (Hays, Widaman, DiMatteo, & Stacy, 1987; Huba, Wingard, & Bentler, 1981; Newcomb, 1986; Newcomb & Bentler, 1986; Newcomb & Harlow, 1986; Stacy, Widaman, Hays, & DiMatteo, 1985).

Sample sizes reported in the 72 reviewed articles were typically large enough to justify the use of asymptotically normal estimators. The median sample size was 198, with a range of 40–8,650. One fourth of the articles used samples of more than 500 observations. Nevertheless, a substantial number of applications (16) involved sample sizes ($n < 100$) that are generally regarded as too small for estimation procedures that require assumptions about population distributions.

Assessment of Fit

The average number of global goodness of fit statistics reported in the 72 articles was 2.2 ($SD = 1.3$). Most of the articles (91.7%) reported a chi-square statistic, 44.4% reported difference chi-squares (involving competing models), 30.6% reported

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3 Sample sizes were averaged if multiple different-sized samples were reported in a single article.
the normed fit index (A), 19.4% reported the ratio of chi-square to degrees of freedom, 12.5% reported the non-normed fit index (p), 12.5% reported the goodness of fit statistic computed by the LISREL computer program, and 11.1% reported the RMR.

From these results it appears that the majority of published applications routinely evaluated the global fit of their models using the chi-square statistic; relatively few articles reported other, perhaps more informative, measures of fit.

Very few articles addressed the fit of individual structural equations. Of the 43 articles in which complete structural models were tested, only 3 reported the squared multiple correlation coefficients.

Most of the 72 articles reported some or all of the estimated parameters (30 gave some of the values, 37 gave all of the values). However, only 41 articles provided t statistics, standard errors, or significance levels for estimated parameters.

An analysis reported by Bagozzi and Burnkrant (1979) is useful for illustrating the interpretational difficulties that arise when individual parameters in a model are not carefully scrutinized. These investigators reanalyzed Fishbein and Ajzen’s (1974) data to test the construct validity of a distinction between affective and cognitive components of attitude (Rosenberg & Hovland, 1960). In this structural model (shown in Figure 3A), two correlated LVs (affective and cognitive components of attitude) were hypothesized as predictors of behavior. Results indicated an acceptable fit of the model to the data, \( \chi^2(11, N = 62) = 13.11, p > .20 \).

The standardized path coefficient for affect (\( \gamma_1 \)) was .65, and for cognition (\( \gamma_2 \)) it was .23. These results led Bagozzi and Burnkrant (1979) to conclude that “the affective component...[is] three times as forceful in its impact on behavior as the cognitive component” (p. 925). This conclusion was not strongly justified because the approximate t values indicated that the path coefficient for cognition (\( \gamma_2 \)) was not reliably different than zero (t = 1.12). The proper way to test the difference is by estimating a model in which the two parameters are constrained to be equal. In this case, the constrained model has \( \chi^2(12, N = 62) = 13.86, p > .20 \). The difference in chi-squares for the constrained and unconstrained models provides a test of the null hypothesis that the two path coefficients are equal. Here, the difference chi-square (1 df) was 0.75, suggesting that the two path coefficients were not, in fact, reliably different.

Bentler and Speckart (1981) also focused on models of the attitude–behavior relation. They used covariance structure modeling to analyze the correlations in a cross-lagged panel design (Kenny, 1975), concluding that “the results...unambiguously support the proposition that attitudes have causal priority over behaviors, as hypothesized” (p. 235). Nevertheless, the attitude–behavior path coefficient had a negative sign in one of the three attitude domains tested. This result creates an interpretational ambiguity because it suggests that behaviors decrease in favorability as attitudes increase in favorability.

**Equivalent Models**

Of the 72 articles reviewed, only 1 acknowledged the existence of a specific equivalent model (Dillon & Kumar, 1985). However, in the majority of instances, plausible and theoretically compelling alternative models could be easily formulated.

The covariance structure model reported by Fredricks and Dossett (1983), and used in the preceding illustration, has a number of plausible and equivalent alternatives (see Figure 2). In this case, any of the paths directed at the BI latent variable can be reversed in any combination without changing the model’s global fit. In addition, any of the nondirected paths involving the three exogenous LVs can be changed to directed paths (in either direction and in any combination) without changing the model’s fit. Thus, one equally fitting and plausible alternative model specifies PB as a direct predictor of SN, BI, AACT, and TB; SN, BI, and AACT as intercorrelated; and BI as a direct predictor of TB. This alternative poses a theoretical challenge to Fredricks and Dossett’s (1983) favored model of the attitude–behavior relation by emphasizing the importance of prior behaviors as a determinant of both attitudes and future behavior.

The analysis reported by Bagozzi and Burnkrant (1979) illustrates how a failure to recognize equivalent models can lead to misleading conclusions. Figure 3A shows a path diagram for the predictive validity model favored by Bagozzi and Burnkrant. However, at least 26 other covariance structure models are equivalent to the one shown in Figure 3A.\(^4\) Two models, in par-
ticular, offer equivalent and theoretically compelling alternatives.

The model shown in Figure 3B reverses the two directed paths, hypothesizing behavior as the predictor of affect and cognition. This model is substantially different from that shown in Figure 3A. Logically, the alternative model indicates that the direction of causal influence cannot be determined by the method of data analysis alone. The alternative model also has theoretical support in self-perception theories of attitude-behavior relations (Bem, 1972), which hypothesize prior behavior as a strong determinant of attitudes.

A third model, shown in Figure 3C, represents the classic tripartite theory of attitude structure (Insko & Schopler, 1967; Katz & Stotland, 1959; Rosenberg & Hovland, 1960), in which the LVs are hypothesized as three intercorrelated components of attitude. This alternative represents a measurement model only and does not include directed relations among the variables. The model shown in Figure 3C makes distinctions among theoretical components of attitude without implying that one component has causal priority over another.

Although the models shown in Figure 3 are equivalent in covariance structure, each leads to a different conclusion about the relation between attitudes and behavior. Further inspection of the individual parameter estimates also reveals differences among the models. As noted previously, the path coefficient for cognition in Figure 3A was not reliably different than zero. However, relations among all three LVs were reliable (all $t$ values $> 4.0$) in the models shown in Figures 3B and 3C. Thus, preference might be given to the latter two models because of their greater parsimony. If a single model is to be favored, the one shown in Figure 3C is most consistent with the cross-sectional nature of the data.

Another example comes from an analysis reported by Pavelchak, Moreland, and Levine (1986). Their favored model is shown in Figure 4. This model takes the same form as those shown in Figure 3, involving three interrelated LVs. This model can be modified by changing the directions of any of the three paths, in any combination, without changing the model's global fit. The three paths can also be changed to correlations (creating a factor model) without changing the fit. One plausible equivalent model reverses the path between beliefs about college and efforts to identify college groups. The latter model is consistent with theories of self-perception and self-justification, in which past behavior is hypothesized as a strong determinant of current attitudes.

Malamuth (1983) tested the covariance structure model shown in Figure 5. According to this model, two exogenous LVs (sexual arousal to rape; attitudes facilitating violence) are hypothesized predictors of one endogenous LV (aggression against women). The exogenous LVs were measured first, followed several days later by measures of the endogenous LV. The primary goal was to develop a prediction equation rather than to support a causal inference. This study demonstrates how covariance structure modeling can be useful in a longitudinal context, but it also illustrates some of the limitations inherent in nonexperimental situations.

The model shown in Figure 5 provided a good fit to the data: $\chi^2 (6, N = 42) = 8.1, p > .05, \Delta = .91$. However, the model also takes the same general form as those shown in Figure 3 and is associated with at least 26 equivalent models. The longitudinal design does not itself rule out the plausibility of most equivalent models. Although it is difficult to justify the reversal of either directed path strictly within the context of this study, the model formed by reversing the two paths may be justified on theoretical grounds. Such a model might have been supported had the order of data collection been reversed. This situation does not detract from the merits of the research because the goal was simply to develop a sound prediction equation. Nevertheless, a degree of ambiguity remains regarding the most appropriate theoretical model for describing interrelations among these variables.

### Nearly Equivalent Models

Two models are defined as equivalent when they share identical fit functions. Although two models may not be formally equivalent, they may be similar enough in fit to be considered within the same class of equivalent models. As an illustration, consider again the attitude-behavior model tested by Fredricks and Dossett (1983), shown in Figure 2. As described in the previous section, this model belongs to a relatively large family of equivalent models. Other models, although not strictly equivalent, also provide a good fit to the data. One such model can be formed by exchanging positions of the AACT and BI latent variables in the path diagram. This model removes the direct effect of BI on TB, hypothesizing AACT and PB as having the only direct effects on TB. The modified model has the same number of free parameters as the original model and also provides a good fit to the data. $\chi^2 (36, N = 236) = 56.3, p > .01$.

All of the interpretational problems that arise with strictly equivalent models apply when nearly equivalent models can be formed. Along these lines, the TETRAD computer program (Glymour et al., 1987) can be helpful in locating many alternative models. However, the TETRAD program is not guaranteed to produce theoretically plausible results.

### Confirming the Consequent

One virtue of confirmatory analysis is that it offers a basis for hypothesis testing. A theoretical model dictates which parameters are to be fixed at some value and which are to be estimated. Such constrained models are supported when they do a good job of reproducing the data. For example, in a confirmatory factor analysis each variable loads on some factors but not on others (loadings are fixed to zero). This stands in contrast to exploratory factor analysis in which all variables are free to load on all factors.

The power of confirmatory factor analysis rests on a priori specification of a theoretical factor structure. If the theories in a particular domain are not refined enough to derive an a priori factor structure, then exploratory factor analysis can be useful in developing the appropriate factor models. New data should then be collected to evaluate these models, using confirmatory factor analysis. This cross-validation strategy of theory development requires use of different data for the exploratory and confirmatory stages of the analysis.

Six of the reviewed articles used the results of exploratory factor analysis to derive a factor structure for confirmatory fac-
of the 72 articles reviewed, 28 (38.9%) reported modifications to an initial model. Among these articles, only 1 based the modifications on explicitly stated theoretical grounds (Vinokur, Schul, & Caplan, 1987); 23 (82.1%) appear to have made modifications strictly on the basis of the data; and 4 (14.3%) used both theory and data to guide the modifications.

**Model Modification**

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The process of model modification (specification searching) can often lead to a model that contains far more estimated parameters than does the original model. An example is a study reported by Newcomb, Huba, and Bentler (1986) in which an initial model was modified (on the basis of the data) by allowing 37 correlations among errors in measurement ($\delta$s). The initial model contained 80 free parameters, but it was rejected on the basis of the chi-square statistic. The modified model (not rejected by the chi-square statistic) contained 37 additional free parameters. Which among the errors in measurement were correlated was not reported, but it is unlikely that the pattern of intercorrelations had a substantive interpretation.

Of the 28 articles reporting model modifications, 3 (10.7%) mentioned the problems associated with specification searches (Bentler & Huba, 1979; Smith, 1982; Tanaka & Huba, 1984) and only 2 (7.1%) used cross-validation to support a modified model (Reisenztein, 1986; Tanaka & Huba, 1984).

**Causal Inferences**

As a method for data analysis, covariance structure modeling does not provide a sufficient basis for inferring causation (cf. Baumrind, 1983). Cross-sectional data clearly do not justify causal inferences. Longitudinal data provide a stronger basis, but other criteria are typically required to firmly support a causal conclusion. Nevertheless, 8 of the 72 articles referred to the method as "causal modeling" or otherwise implied a causal inference in their titles. Twenty-nine of the articles used phrases that implied that causal relations were established (phrases of the form $a$ affects $b$, $c$ has an impact on $d$, $e$ is influenced by $f$, etc.). One example of a strong causal claim is reflected in the previous quotation from Bentler and Speckart's (1981) study of attitude–behavior relations.

Some investigators were sensitive to the limitations of their data: Causal inferences were made in only 11 of the 44 cross-sectional studies, whereas similar inferences were drawn in 13 of the 21 longitudinal studies and in 5 of the 7 experiments.

Baumrind (1983) cogently argued against making strong causal claims on the basis of correlational data. Baumrind's article appeared in print (in the Journal of Personality and Social Psychology) before many of the articles included in the present review were submitted for publication. Nevertheless, it was cited only once (by Hays et al., 1987), even though the problem area addressed by Baumrind (drug use) was directly relevant to other studies (e.g., Stein, Newcomb, & Bentler, 1987). Thus, when it comes to making causal inferences, it would appear that investigators are not as sensitive as they can be to the limitations of their data.

**Summary, Recommendations, and Conclusion**

**Summary**

Covariance structure modeling is commonly used in personality and social psychology. It is a powerful data analytic tool with a variety of useful applications. However, serious problems associated with the method have not been completely acknowledged or addressed in the personality and social psychology literature. Among these problems are (a) potential violations of distributional assumptions, (b) failure to recognize the existence of equivalent models, (c) use of the same data to both derive and confirm models, (d) modification of models without cross-validation, and (e) poorly justified causal inferences.

Other problems are not so readily detected. For example, if the modification history is not described in a published report, it may be difficult or impossible to know how many changes were made to an initial model. It is even possible that the final product of a series of modified models will be presented as the a priori model. Such practices may be common in the pursuit to confirm initial hypotheses (cf. Greenwald, Pratkanis, Leippe, & Baumgardner, 1986).

**Recommendations**

Several recommendations may help to avoid some of the deficiencies observed in previously published applications of covariance structure modeling.

The data should be inspected for potential violations of the multivariate normality assumption. The EQS computer program (Bentler, 1985) is especially helpful in this regard. Distributional assumptions are likely to be violated when variables are strongly skewed or when extreme outliers are present. These situations should be acknowledged in published reports. In such cases, the investigator is advised to pursue one of the distribution-free methods for parameter estimation.

Authors must make every effort to identify equivalent models and to discuss whether such models offer plausible representations of the data. In many instances, it may be possible to eliminate equivalent models on theoretical or logical grounds. Other features of the data-collection context (e.g., an experimental design or supplemental data) may make some equivalent models implausible. Editors and reviewers must also be vigilant in detecting the possibility of equivalent models.

Cross-validation should be conducted whenever an initial model is modified on the basis of the data (see Cudeck & Browne, 1983). A prudent procedure is to routinely divide the original sample into two parts: a derivation sample and a cross-validation sample. The derivation sample can be used to fit the initial model and to derive modifications of it. Once a favored model is found, its fit can be assessed by using (different) data from the cross-validation sample. Editors and reviewers should be skeptical of models that are "confirmed" with the same data as were used to make modifications.

Published accounts of covariance structure modeling should provide enough details of the analysis to permit replication by other investigators. It is often the case that a model's estimated parameters are not made explicit. This is especially common when a model has been modified or when many of the measurement errors ($\delta$s or $\epsilon$s) are correlated. Unless all of a model's free parameters are clearly defined, the reader has no way of knowing the precise model being fit.

Whenever feasible, the data (correlations or covariances) should be provided as part of the published report. If the number of data matrices or space limitations prevent publication of the data, authors should make the data readily available. This practice helps other investigators to replicate the analysis and to fit rival models to the data. It can also be useful in evaluating...
differences in solutions obtained among different computer programs.

Conclusion

Covariance structure modeling is rapidly becoming a popular data analytic tool. Indeed, McGuire (1973) anticipated its popularity as "the new methodology where correlation can indicate causation" (p. 454). When used in suitable contexts, and with appropriate cautions in mind, covariance structure modeling can be useful in making sense of complex interrelations in multivariate data sets. However, the power attributed to covariance structure modeling is often overstated, as when McGuire (1985) described it as being used "to detect the links and directions of causal flow" (p. 238). Recent applications of covariance structure modeling have indeed created cause for concern. It is hoped that the issues identified here will be properly addressed in future applications.

References


Appendix

Publications Reporting the Use of Covariance Structure Analysis (1979–1987)

Journal of Experimental Social Psychology


Journal of Personality and Social Psychology


**Personality and Social Psychology Bulletin**


**Psychological Review**


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